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Scalar and joint velocity–scalar PDF modelling of near-wall turbulent heat transfer

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Abstract

The temperature field in a heated turbulent flow is considered as a dynamically passive scalar. The probability density function (PDF) method with down to the wall integration is explored and new modelling proposals are put forward, including the explicit account for the molecular transport terms. Two variants of the approach are considered: first, the scalar PDF method with the use of externally-provided turbulence statistics; and second, the joint (stand-alone) velocity–scalar PDF method where a near-wall model for dynamical variables is coupled with a model for temperature. The closure proposals are formulated in the Lagrangian setting and resulting stochastic evolution equations are solved with a Monte Carlo method. The near-wall region of a heated channel flow is taken as a validation case; the second-order thermal statistics are of a particular interest. The PDF computation results agree reasonably with available DNS data. The sensitivity of results to the molecular Prandtl number and to the thermal wall boundary condition is accounted for.
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1. Introduction

Turbulent heat transfer in wall-bounded flows is a topic of considerable scientific and practical interest. The issue remains open due to the engineering relevance of heat transfer problems and the inherent difficulties in physically sound turbulence modelling. A detailed picture includes a complex evolution of near-wall vortical flow structures and the unsteady heat conduction in the wall material. At the level of statistical averages, the temperature variance, the turbulent heat flux, and the near-wall temperature spectra are of direct engineering interest. In particular, a motivation to undertake the present work originated from some industrial situations related to the conjugate heat transfer problems where both thermomechanics of fluids and thermal stresses in solids come into play in a coupled way. In those situations, fairly detailed estimation of turbulence and thermal statistics in the near-wall region is desirable for optimal design purposes and avoiding thermal fatigue of wall material.

Numerical approach to near-wall turbulent flows with heat transfer is mostly undertaken in the Eulerian description. The statistical approach with the Reynolds-averaged Navier–Stokes (RANS) equations remains a standard engineering tool providing at least some of the desired thermal statistics, despite notorious difficulties related to near-wall treatment (cf. reviews of Launder, 1988 and Nagano, 2002). On the other hand, there is a growing interest in DNS and LES studies; they are still computationally expensive yet otherwise irreplaceable in discovering the detailed flow physics, and provide valuable reference data for the assessment and improvements of statistical models. The DNS is extremely useful for theoretical studies, yet impracticable for engineering problems. The LES is a very promising approach in the latter respect, yet still somewhat problematic in the near-wall modelling context (Piomelli and Balaras, 2002). We note that hybrid LES approaches with statistical models applied in the near-wall region

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have been put forward recently (Hamba, 2002; Dahlström and Davidson, 2003).

Although the aim of providing detailed turbulence and thermal statistics in the near-wall region could a priori be achieved using DNS or (with some reserves) LES, a realistic aim for routine analysis and design purposes is rather to get some limited information at a lower cost, using the statistical description. To pursue this goal, the approach followed in the present work is the probability density function (PDF) method (cf. Pope, 2000). As an alternative statistical approach, the one-point PDF method is able to give more detailed information on the flow and thermal fields than the RANS, yet at a lower computational effort than the LES.

Two variants of the approach are considered separately in the paper. First, the computed scalar PDF method for the temperature where the velocity statistics have to be externally provided. Usually, it is done by an Eulerian solver coupled to the PDF computation; here, we have taken the Eulerian velocity statistics from the available DNS data. The computations with the scalar PDF approach using a simplified near-wall scalar model have been reported recently (Pozorski et al., 2003b); the model is further developed here (Section 3). As a second variant, the joint velocity–scalar PDF approach is considered. It can provide one-point turbulence statistics of any order, including mixed temperature–velocity moments, as well as Lagrangian autocorrelation functions. In the stand-alone approach, no coupling to the Eulerian flow solver is necessary; an evolution equation for the one-point joint PDF of velocity, dissipation rate, and temperature is formulated. It is closed and solved in the Lagrangian setting, with the use of stochastic particles (notional fluid elements). We report some further developments in the joint velocity–scalar PDF method (Section 4) as a follow up of our recent studies. To model the wall-bounded turbulent flows with the stand-alone velocity PDF method, the authors used both the wall-function approach (Minier and Pozorski, 1999) and the integration of the conservation equations through the viscous sublayer down to the wall (Wacławczyk et al., 2004). For flows with the temperature field, the joint PDF of velocity and temperature is introduced. The joint velocity–scalar PDF method has been developed with the so-called law of the wall, bridging through the viscous sublayer (Pozorski et al., 2003a). Yet, the results obtained in the wall function approach are not fully satisfactory for lower Reynolds numbers. Moreover, if detailed near-wall thermal statistics are needed, the down to the wall integration of flow equations has to be performed (and this remains quite expensive in terms of computational work), rather than using the wall function approach. Despite first efforts in this direction (Mazumder and Modest, 1997), such models have not been well studied so far.

The first aim of the paper is to report new developments regarding the near-wall temperature modelling in both variants of the PDF method considered here. The second aim is to compare and validate them in the case of the fully-developed channel flow with constant heat flux. Another aim is to discuss the sensitivity of near-wall thermal fluctuations to the boundary conditions (isothermal or isoflux) and the molecular Prandtl number. The results for thermal statistics will be compared with the available DNS reference data.

2. Mean equations

The energy equation to be considered in the paper is formulated with suitable simplifying assumptions of no internal heat sources (negligible temperature rise due to the dissipation of kinetic energy of the flow) and constant fluid properties (the density ρ, the kinematic viscosity ν, and the molecular thermal diffusivity α). It is written in the form of the advection–diffusion equation for the instantaneous temperature:

$$\frac{\partial T}{\partial t} + U_j \frac{\partial T}{\partial x_j} = \alpha \frac{\partial^2 T}{\partial x_j \partial x_j}. \tag{1}$$

Moreover, with constant fluid properties and resulting negligible buoyancy effect, the momentum equation is decoupled from the energy equation and the temperature can be considered as a passive scalar variable.

The starting point of the statistical description of turbulent flows is the Reynolds decomposition of relevant variables; for velocity and temperature we write $U_i = \langle U_i \rangle + u_i$, and $T = \langle T \rangle + \theta$, respectively. The mean of Eq. (1) takes the form

$$\frac{\partial \langle T \rangle}{\partial t} + \langle U_j \rangle \frac{\partial \langle T \rangle}{\partial x_j} = -\frac{\partial \langle u_j \theta \rangle}{\partial x_j} + \alpha \frac{\partial^2 \langle T \rangle}{\partial x_j \partial x_j}. \tag{2}$$

At the lowest closure level, effective (turbulent) transport coefficients ν_t and α_t are introduced in the standard way from the Boussinesq assumption:

$$\begin{aligned} \langle u_i u_j \rangle &= -\nu_t \left(\frac{\partial \langle U_i \rangle}{\partial x_j} + \frac{\partial \langle U_j \rangle}{\partial x_i} \right) + \frac{2}{3} k \delta_{ij}, \\ \langle u_j \theta \rangle &= -\alpha_t \frac{\partial \langle T \rangle}{\partial x_j}, \end{aligned} \tag{3}$$

where $k = \langle u_i u_i \rangle / 2$ is the turbulent kinetic energy. The molecular and turbulent Prandtl number are respectively defined as

$$Pr = \frac{\nu}{\alpha}, \quad Pr_t = \frac{\nu_t}{\alpha_t}. \tag{4}$$

Hence, at the lowest closure level, Eq. (2) is written as

$$\frac{\partial \langle T \rangle}{\partial t} + \langle U_j \rangle \frac{\partial \langle T \rangle}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\alpha + \frac{\nu_t}{Pr_t} \right) \frac{\partial \langle T \rangle}{\partial x_j} \right]. \tag{5}$$

In a known mean velocity field, Eq. (5) can be solved for given Pr_t and ν_t. The former is often assumed $Pr_t = 1$ from the so-called Reynolds analogy. The latter can be determined either from algebraic mixing-length type formulae or from eddy viscosity models. In the paper (the scalar PDF part), instead of the standard expression $\nu_t = C_\mu f_\mu k^2/\epsilon$ we apply a proposal of Durbin (1991), (cf. Pope, 2000) for ν_t in the near-wall layer, confirmed by the DNS:

$$\nu_t = C'_\mu \frac{\langle v^2\rangle k}{\epsilon}, \tag{6}$$

where $C'_\mu = 0.22$. To lend support to this formula we note that the near-wall transport is mainly due to the wall-normal velocity fluctuations; because of the kinematic wall-blocking effect, $\langle v^2\rangle$ is damped more than the other diagonal components of the turbulent stress tensor.

The exact equation for the fluctuating temperature variance $\langle\theta^2\rangle$ reads (cf. Nagano, 2002):

$$\frac{\partial\langle\theta^2\rangle}{\partial t} + \langle U_j\rangle\frac{\partial\langle\theta^2\rangle}{\partial x_j} = \alpha\frac{\partial^2\langle\theta^2\rangle}{\partial x_j\partial x_j} - \frac{\partial\langle u_j\theta^2\rangle}{\partial x_j} + 2\mathscr{P}_\theta - 2\epsilon_\theta. \tag{7}$$

The first and second RHS terms of Eq. (7) represent the molecular and turbulent diffusion; $\mathscr{P}_\theta$ and ϵ_θ respectively stand for the production and destruction rate of $k_\theta = \langle\theta^2\rangle/2$:

$$\mathscr{P}_\theta = -\langle u_j\theta\rangle\frac{\partial\langle T\rangle}{\partial x_j}, \qquad \epsilon_\theta = \alpha\left\langle\frac{\partial\theta}{\partial x_j}\frac{\partial\theta}{\partial x_j}\right\rangle. \tag{8}$$

3. Near-wall scalar PDF approach

3.1. Scalar PDF transport equation

Arguably, the near-wall thermal statistics can be studied first with the scalar PDF approach, and some valuable insight can hopefully be gained this way. The disadvantage of this formulation is that the gradient diffusion hypothesis for turbulent transport is used, yet the computational cost is considerably reduced. The scalar PDF methods were historically developed first, and used in coupling with Eulerian flow solvers, mainly for turbulent combustion problems (Dopazo, 1994; Jones, 2002). Following the preliminary studies presented in Pozorski et al. (2003b), the near-wall scalar PDF approach is further developed in the present paper. The necessary external input to the model, i.e. moments of the velocity field, will be taken from available DNS data of the heated turbulent channel flow.

The one-point PDF of a scalar variable (here: temperature) in turbulent flow is introduced as $f = f_{T\mathbf{X}}(\Theta, \mathbf{x}, t)$; the sampling space variables $\mathbf{x}$ and Θ respectively correspond to the location $\mathbf{X}$ and the instantaneous temperature T that are random variables or, more precisely, stochastic processes when their time evolution is followed. Provided that moments of the velocity field are given, the exact transport equation for f is readily derived from the instantaneous temperature equation (Eq. (1)), using standard methods [cf. Pope (2000), Eq. (12.327)] and further rearranged as follows (symbol $\langle\cdot|\cdot\rangle$ denotes the conditional average)

$$\frac{\partial f}{\partial t} + \frac{\partial f\langle U_k\rangle}{\partial x_k} + \frac{\partial}{\partial x_k}[\langle u_k|T=\Theta\rangle f] = \alpha\frac{\partial^2 f}{\partial x_k\partial x_k} - \frac{\partial^2}{\partial\Theta^2}\left[\left\langle\alpha\frac{\partial T}{\partial x_k}\frac{\partial T}{\partial x_k}\middle|T=\Theta\right\rangle f\right], \tag{9}$$

to separate the molecular diffusion term in the physical space, i.e. the first RHS term that is exact and requires no closure. To close the last LHS term of (9), the gradient hypothesis for turbulent transport is applied (Pope, 2000):

$$\langle u_k|T=\Theta\rangle f = -\alpha_t\partial f/\partial x_k. \tag{10}$$

The last RHS term of (9) presents a major difficulty for modelling. It is further decomposed as

$$\frac{\partial^2}{\partial\Theta^2}\left[\left\langle\alpha\frac{\partial T}{\partial x_k}\frac{\partial T}{\partial x_k}\middle|T=\Theta\right\rangle f\right] = \frac{\partial^2}{\partial\Theta^2}\left[\alpha\frac{\partial\langle T\rangle}{\partial x_k}\frac{\partial\langle T\rangle}{\partial x_k}f\right] + \frac{\partial^2}{\partial\Theta^2}\left[\left\langle\alpha\frac{\partial\theta}{\partial x_k}\frac{\partial\theta}{\partial x_k}\middle|T=\Theta\right\rangle f\right]. \tag{11}$$

The second expression on the RHS of (11) is closed with a scalar mixing model. Although many such models have been proposed to date, none of them is fully satisfactory (cf. Dopazo, 1994 or Jones, 2002). In a previous paper (Pozorski et al., 2003b), the scalar mixing model accounted for the whole sink term, i.e. the LHS of Eq. (11). This is tantamount to neglecting the near-wall molecular transport effects. Here, a modification consists in applying a scalar mixing model in the usual way (to the fluctuating part only), and a proposal for the mean temperature part is put forward. Let us introduce first some useful quantities. The RHS of (11) includes the mean scalar dissipation, denoted by $\epsilon_\theta^{\mathrm{M}}$:

$$\epsilon_\theta^{\mathrm{M}} = \alpha\frac{\partial\langle T\rangle}{\partial x_k}\frac{\partial\langle T\rangle}{\partial x_k}, \tag{12}$$

and the dissipation rate of temperature fluctuations ϵ_θ, cf. Eq. (8); it will further be assumed that the scalar dissipation ϵ_θ is independent of temperature. The LHS of (11) includes the complete (mean and turbulent) scalar dissipation rate $\epsilon_\theta^{\mathrm{C}}$:

$$\epsilon_\theta^{\mathrm{C}} = \alpha\left\langle\frac{\partial T}{\partial x_k}\frac{\partial T}{\partial x_k}\right\rangle = \epsilon_\theta^{\mathrm{M}} + \epsilon_\theta. \tag{13}$$

Consequently, with the use of Eqs. (10)–(12), Eq. (9) takes the form

$$\frac{\partial f}{\partial t}+\frac{\partial}{\partial x_k}\left[\left(\langle U_k\rangle+\frac{\partial \alpha_t}{\partial x_k}\right)f\right]=-\frac{\partial^2}{\partial \Theta^2}\left[\left\langle \alpha\frac{\partial\theta}{\partial x_k}\frac{\partial\theta}{\partial x_k}\middle| T=\Theta\right\rangle f\right]+\frac{\partial^2(\alpha+\alpha_t)f}{\partial x_k\partial x_k}-\epsilon_\theta^M\frac{\partial^2 f}{\partial\Theta^2}. \quad (14)$$

It is interesting to estimate the level of ϵ_θ^M and ϵ_θ at the wall ($y\to 0$). The mean temperature in the fully developed channel flow changes as $|\langle T\rangle - T_w|/\theta_* = Pr\ u_* y/\nu$ near the wall. After a suitable normalisation by the friction temperature θ_*, the friction velocity u_*, and the viscous length scale $\delta_\nu=\nu/u_*$, this yields the non-dimensional mean dissipation $(\epsilon_\theta^M)^+ = Pr$ in the near-wall region. On the other hand, the DNS data of Kawamura et al. (1998) at $Re_\tau = 180$ (based on u_* and the channel half-width) and $Pr=1$ for the turbulent dissipation of thermal fluctuations yield the wall maximum of $\epsilon_\theta^+\approx 0.15$. Thus, the mean dissipation is dominant in the conductive viscous sublayer. In Eq. (14), two physically distinct facets of the molecular diffusivity coefficient α are evidenced: in the first RHS term it enters the sink term of the thermal fluctuation variance at the high-wavenumber end of the temperature spectrum, while in the remaining RHS terms α also enters the diffusion and mean dissipation terms (important in the near-wall layer but otherwise negligible for high-*Re* regions). In the majority of turbulent combustion problems, the near-wall regions are less important in modelling, since the reaction mostly occurs in the core of the flow. For this reason, when proposing turbulent mixing models, attention has been focused on the first RHS term of Eq. (14) while the remaining molecular terms have usually been neglected.

3.2. Scalar mixing model

Modelling of the temperature as a scalar variable involves a difficulty due to the mixing term that remains unclosed in one-point approaches. Arguably the simplest closure, used in the paper, is the IEM (interaction by exchange with the mean) model of the relaxation type (Dopazo, 1994):

$$\frac{\partial f}{\partial t}=\frac{\partial}{\partial\Theta}\left[\frac{1}{\tau_\phi}(\Theta-\langle T\rangle)f\right]\Longleftrightarrow \mathrm{d}T=-\frac{T-\langle T\rangle}{\tau_\phi}\,\mathrm{d}t, \quad (15)$$

with the scalar mixing time scale related to the turbulent scale:

$$\tau_\phi=\frac{2}{C_\phi}\frac{k}{\epsilon}, \quad (16)$$

where $C_\phi=2$ is the model constant.

As argued previously, the molecular transport and dissipation terms can not be neglected in the near-wall region. The mean dissipation ϵ_θ^M that becomes dominant in the viscous sublayer can be accounted for in a way analogous to that proposed for the mean dissipation term in the momentum equation (Wacławczyk et al., 2004). Namely, the last RHS term in (14) represents the antidiffusion in the phase space of scalar variable. Physically speaking, this term is non-local and has a similar character as the first RHS term, i.e. the turbulent scalar mixing term. Hence, the modelling difficulty is of the same origin: there is no exact particle representation of the antidiffusion term $-\partial^2 f/\partial\Theta^2$ in the one-point PDF method; we recall that the second RHS term, i.e. the diffusion term $\partial^2 f/\partial x_k^2$ can be represented exactly through the Wiener process. The first RHS term of Eq. (14), i.e. the scalar turbulent mixing can be modelled through the Lagrangian equation with the IEM (cf. Eq. (15)). Our new proposal consists in modelling the last RHS term of Eq. (14), i.e. the mean scalar dissipation term, by a relaxation towards the local mean temperature, analogously to the IEM mixing model

$$-\epsilon_\theta^M\frac{\partial^2 f}{\partial\Theta^2}\Rightarrow -\frac{1}{\tau_M}(T-\langle T\rangle)\,\mathrm{d}t \quad (17)$$

with the value of τ_M (a time scale) determined below. Finally, the evolution equation for the PDF, Eq. (14), closed with expressions (15) and (17) becomes:

$$\frac{\partial f}{\partial t}+\frac{\partial}{\partial x_k}\left[\left(\langle U_k\rangle+\frac{\partial\alpha_t}{\partial x_k}\right)f\right]=\frac{\partial^2(\alpha+\alpha_t)f}{\partial x_k\partial x_k}+\frac{\partial}{\partial\Theta}\left[\left(\frac{1}{\tau_\phi}+\frac{1}{\tau_M}\right)(\Theta-\langle T\rangle)f\right]. \quad (18)$$

This equation after suitable integration yields the mean temperature balance (5) and a certain temperature variance equation:

$$\frac{\partial\langle\theta^2\rangle}{\partial t}+\langle U_i\rangle\frac{\partial\langle\theta^2\rangle}{\partial x_i}=\frac{\partial}{\partial x_i}\left[(\alpha+\alpha_t)\frac{\partial\langle\theta^2\rangle}{\partial x_i}\right]+2(\alpha+\alpha_t)\frac{\partial\langle T\rangle}{\partial x_i}\frac{\partial\langle T\rangle}{\partial x_i}-2\left(\frac{1}{\tau_\phi}+\frac{1}{\tau_M}\right)\langle\theta^2\rangle. \quad (19)$$

The exact evolution equation for $\langle\theta^2\rangle$, Eq. (7), contains only the dissipation ϵ_θ connected with the turbulent fluctuations (and not ϵ_θ^M). Consequently, τ_M is determined from the requirement

$$\frac{1}{\tau_M}\langle\theta^2\rangle=\alpha\frac{\partial\langle T\rangle}{\partial x_k}\frac{\partial\langle T\rangle}{\partial x_k}, \quad (20)$$

to assure that the molecular transport term in the scalar variance Eq. (19) is correct. Finally, for the variance we have:

$$\frac{\partial\langle\theta^2\rangle}{\partial t}+\langle U_k\rangle\frac{\partial\langle\theta^2\rangle}{\partial x_k}=\frac{\partial}{\partial x_k}(\alpha+\alpha_t)\frac{\partial\langle\theta^2\rangle}{\partial x_k}+2\alpha_t\frac{\partial\langle T\rangle}{\partial x_k}\frac{\partial\langle T\rangle}{\partial x_k}-\frac{2}{\tau_\phi}\langle\theta^2\rangle; \quad (21)$$

the RHS terms model the diffusion, production and destruction, respectively (cf. the exact formula (7)).

3.3. Near-wall modelling: estimation of external parameters

We recall here that the turbulent thermal diffusivity is estimated from $\alpha_t = \nu_t/Pr_t$. Given the expression for the turbulent viscosity ν_t, Eq. (6), the statement of the turbulent Prandtl number is necessary. Kays (1994) offers a comprehensive survey of the available data on Pr_t. Traditional approach to this problem goes through the assumption about the similarity of dynamical and thermal near-wall fields, generally valid for the molecular Prandtl number close to 1 and for similar boundary conditions for the two fields. Consequently, for the computations with isothermal wall b.c. we will take $Pr_t = 1$. Nevertheless, a modification is necessary for the isoflux b.c. with non-zero temperature variance at the wall; these conditions are qualitatively different from those for velocity. The correct near-wall scaling for turbulent thermal diffusivity is $\alpha_t = \mathcal{O}(y^2)$, resulting from $\langle v\theta\rangle = \alpha_t(\partial\langle T\rangle/\partial y)$ for $v = \mathcal{O}(y^2)$ and $\theta = \mathcal{O}(1)$. Hence for the isoflux b.c. the result is $Pr_t = \mathcal{O}(y)$ near the wall, supported by the DNS of the boundary layer (Kong et al., 2000). Also, identical scaling follows from the unsteady structural model of Kasagi et al. (1989) and from the theoretical expression for $Pr_t = Pr_t(Pr, y, K)$ derived by Geshev (1978) for the conjugate heat transfer (for finite values of the thermal activity ratio K) under certain assumptions (including those for the form of the two-point velocity correlations in the near-wall region). Consequently, for the computations with isoflux wall b.c. we will use a tentative formula

$$Pr_t(y^+) = 1 - \exp\left(-\frac{y^+}{5.0}\right), \tag{22}$$

corresponding roughly to the profile reported by the DNS of Lu and Hetsroni (1995) at $Re_\tau = 184$ and $Pr = 0.7$. Regarding the dependence of Pr_t on Pr, several DNS studies for channel flow with isothermal b.c. predict a rather weak increase of Pr_t with decreasing Pr, the trend being more pronounced for $Pr < 0.1$ (Kim and Moin, 1989; Kawamura et al., 1999). We note that the analytical expression of Geshev (1978) predicts just the contrary (increase of Pr_t with increasing Pr), thereby calling into question some of the assumptions behind his reasoning.

Concerning the scalar mixing time scale τ_ϕ, the high-Re turbulent time scale k/ϵ used in Eq. (16) is modified in the near-wall region to account for the Kolmogorov microscale, following the proposal of Durbin (1993). Consequently, for the isothermal wall b.c., Eq. (16) is replaced by

$$\tau_\phi = \frac{2}{C_\phi}\max\left\{\frac{k}{\epsilon}, C_T\left(\frac{\nu}{\epsilon}\right)^{1/2}\right\}, \tag{23}$$

with $C_T = 6.0$.

The underlying assumption behind Eq. (23) is the similarity of velocity and temperature fields; it basically implies the proportionality of dynamical and thermal time scales. This assumption is probably justified only for Pr not too far from unity. What is of utmost importance for us here, is that this assumption is questionable for boundary conditions of differing nature (as in the case of the isoflux no-slip wall) where the velocity and temperature fields in the near-wall region definitely differ. As already noticed from the comparison of the exact equation for the temperature variance (Eq. (7)), with its modelled counterpart (Eq. (21)), the destruction rate of temperature fluctuations is represented as $\epsilon_\theta = \langle\theta^2\rangle/\tau_\phi$ where τ_ϕ is definitely the scalar time scale and the assumption of its proportionality to the turbulent time scale k/ϵ (Eq. (16)), breaks down in the near-wall region of the heated flow with the isoflux boundary conditions. Let us introduce the ratio of scalar to turbulent time scales

$$R = \frac{\langle\theta^2\rangle/2\epsilon_\theta}{k/\epsilon}. \tag{24}$$

It is readily noticed that due to $\langle\theta^2\rangle_w \neq 0$ the ratio R tends to infinity at the wall. In other words, from the near-wall Taylor series expansion we have $1/R = \mathcal{O}(y^2)$. A possibility that comes to mind is to link the near-wall behaviour of R to that of Pr_t, i.e.

$$\frac{1}{R} \sim Pr_t^2, \tag{25}$$

for $y \to 0$. On the other hand, the limit to be respected far from the wall is $R = 1/C_\phi$. Consequently, in the computations with the isoflux b.c. we have taken $\tau_\phi = (2/C_\phi)Pr_t^{-2}k/\epsilon$; with this prescription, the scalar mixing time scale is no longer proportional to the turbulent time scale in the near-wall region, and a non-zero value of τ_ϕ results in the wall limit.

For Pr that are not close to unity, a more general formulation is needed that would account for various regimes and scales in turbulent flows with scalars. All this said, we have yet to state that the goal of finding a possibly simple, but physically sound, analytical expression that would describe the dependence of both Pr_t and R on the molecular Prandtl number seems to be an elusive one.

3.4. Boundary conditions

Suitable boundary conditions supplement the closed PDF transport equation. We state them both in terms of the density function $f(\Theta, \mathbf{x}, t)$ and in terms of the variables $(T, \mathbf{X})$ associated with stochastic particles, due to

the equivalence of the Fokker–Planck equation and the corresponding stochastic differential equation (cf. Section 3.5 below).

First, natural boundary conditions for the unbounded scalar variable in the Θ-space are $f(\Theta = \pm\infty, \mathbf{x}, t) = 0$; yet, finite limits are usually set up by the wall temperature extremes. Possible b.c. in physical space include the wall boundary condition at $\mathbf{x} = \mathbf{x}_w$:

$$f(\Theta, \mathbf{x} = \mathbf{x}_w, t) = f_w(\Theta, \mathbf{x}_w, t), \tag{26}$$

where f_w denotes the PDF (in general: unsteady) of the wall temperature. For the isothermal wall $f_w(\Theta, \mathbf{x}_w, t) = \delta(\Theta - T_w(\mathbf{x}_w, t))$. For the isoflux boundary (prescribed wall heat flux), the statement of the wall boundary condition is less clear. The symmetry condition

$$\frac{\partial f}{\partial y}(\Theta, y = 0, t) = 0, \tag{27}$$

looks adequate in a particular case of the adiabatic wall. In general, i.e. for non-zero wall heat flux implying non-zero mean temperature gradient at the wall, the symmetry condition should rather be put in terms of the PDF of the fluctuating temperature, denoted by $g(\theta, \mathbf{x}, t)$:

$$\frac{\partial g}{\partial y}(\theta, y = 0, t) = 0. \tag{28}$$

For the conjugate heat transfer, the wall level of thermal fluctuations cannot be given in advance. It results from a coupled problem where the temperature field inside the wall material is to be computed.

For the case of a fully-developed turbulent channel flow with both walls heated, the symmetry condition is applied at the centerline $y/H = 1$

$$\frac{\partial f}{\partial y}(\Theta, y = H, t) = 0. \tag{29}$$

The thermal boundary conditions formulated in terms of stochastic particles are stated as follows. First, at the centerline (CL) of the channel heated equally at both walls the symmetry condition takes the form:

$$T_{in} = T_{out}. \tag{30}$$

For both isothermal and isoflux cases, the constraints for the statistics (or their gradients) at the boundary readily follow from the pairwise averaging where only an outgoing particle and its incoming "peer" are considered. The b.c. (30) imposes the zero mean gradient at the boundary; moreover, zero gradient of the variance results there:

$$\left.\frac{\partial \langle T\rangle}{\partial n}\right|_{CL} = 0, \quad \left.\frac{\partial \langle \theta^2\rangle}{\partial n}\right|_{CL} = 0;$$

yet, both $\langle T\rangle_{CL} \neq 0$ and $\langle \theta^2\rangle_{CL} \neq 0$ are results of the simulation and cannot be imposed beforehand.

Since molecular transport effects are modelled by the random walk of particles in physical space, some particles cross the boundary, say $y_w = 0$. Then, they are reflected from it, with $y_{in} = |y_{out}|$. The boundary condition imposed at the isothermal wall of temperature T_w for particles that are reflected from the wall, either truly or in the sense of reflected Brownian walk (Wacławczyk et al., 2004), writes

$$T_{in} = T_w. \tag{31}$$

The boundary condition imposed at the isoflux wall is

$$T_{in} = 2T_w - T_{out};$$

this type of boundary condition imposes a constant mean value T_w at the boundary; zero gradient of the variance results at the wall:

$$\langle T\rangle_w = T_w, \quad \left.\frac{\partial \langle \theta^2\rangle}{\partial n}\right|_w = 0.$$

We note that in Eulerian RANS there is a need to impose the level of the temperature variance at the wall. For example, in the R_{ij}–ϵ–k_θ–ϵ_θ model of Sommer et al. (1994) the value of $\langle \theta^2\rangle_w$ is externally provided; for the purpose, those authors use the boundary condition from an analytical formula (Polyakov, 1974). On the other hand, in the PDF approach there is no such problem, since for the isoflux case the wall level of temperature fluctuations makes part of the solution.

3.5. Monte Carlo solution method

The PDF Eq. (18), supplemented with the formulae for $\alpha_t = \nu_t/Pr_t$ and τ_ϕ, (cf. Section 3.3) and suitable boundary conditions (detailed in Section 3.4), can basically be solved with the finite-difference method. Yet, we find it more practical to solve the PDF transport equation using the stochastic particle method. Here, $\nu_t/\nu = C'_\mu \langle v^{+2}\rangle k^+/\epsilon^+$ and $\tau^+_\phi = (2/C_\phi)k^+/\epsilon^+$ are given functions of y^+. In our computations, both functions serve as input data. We have reconstructed the two profiles from the DNS data of Moser et al. (1999) at $Re_\tau = 180$ and 395; resulting plots of k^+/ϵ^+ and ν_t/ν can be found in the paper of Pozorski et al. (2003b). The system of stochastic differential equations equivalent to the Fokker–Planck Eq. (18) is:

$$\mathrm{d}X_i = \langle U_i\rangle \,\mathrm{d}t + \frac{\partial \alpha_t}{\partial x_i}\,\mathrm{d}t + \sqrt{2(\alpha + \alpha_t)}\,\mathrm{d}W_i, \tag{32}$$

$$\mathrm{d}T = -\left(\frac{1}{\tau_\phi} + \frac{1}{\tau_M}\right)(T - \langle T\rangle)\,\mathrm{d}t. \tag{33}$$

In practice, the system is rather solved in the non-dimensional setting, after suitable scales (δ_ν, u_*, and θ_*) are introduced.

In the PDF formulation for the scalar variable, as opposed to the joint PDF model for position and

velocity, no flow dynamics is solved; so there is no a priori guarantee that the distribution of stochastic particles will remain uniform in physical space. For the incompressible case considered in the paper, this automatically guarantees the constant fluid density. This problem has been addressed by Pozorski et al. (2003b); it can be proved theoretically that initially uniformly distributed particles will remain so in time; moreover, numerical tests with a finite number of particles per computational cell are in line with the predictions.

We recall the near-wall Taylor series expansion of turbulent quantities: $k = \mathcal{O}(y^2)$, $\epsilon = \mathcal{O}(1)$, and for the isothermal wall $\theta = \mathcal{O}(y)$. Accounting for Eq. (16), it is readily seen that the expression θ/τ_ϕ in the scalar mixing models behaves as y^{-1} in the limit of $y \to 0$. Consequently, the simplest first-order explicit (Euler) scheme for the SDEs can not be applied in case of down to the wall integration. Instead, the exponential scheme is introduced. It is based on the analytical solution over a time step of the scalar evolution equation with coefficients frozen at their values at the beginning of the time step. Moreover, another difficulty is due to a large span of the relevant time scales (more than two orders of magnitude). This results in a large number of time steps necessary to achieve convergence of computation in the whole domain.

4. Near-wall joint velocity–scalar PDF approach

Another approach considered here is the joint velocity–scalar PDF formulation (with stochastic differential equations solved for particle location, velocity, and temperature). For the purpose, a recent near-wall velocity PDF model of the authors (Wacławczyk et al., 2004) is extended to account for temperature. In the resulting near-wall PDF model with the explicit solution of the flow in the viscous sublayer there is no recourse to the gradient transport hypothesis. Moreover, such an approach makes it possible to retrieve the turbulent heat flux vector and other joint velocity–scalar statistics. Complete models with down-to-the-wall integration are much more computationally expensive, but otherwise they prove to be physically sound in regions of adverse pressure gradients, separation, or recirculation. Here, we apply the one-point near-wall PDF model for position, velocity and the turbulent frequency ω (the inverse of the time scale, $\langle\omega\rangle = \epsilon/k$). Stochastic particles (notional fluid elements) move in physical space by advection and the Brownian motion, their velocity changes according to the Langevin model (determined by a drift matrix G_{ij} and a diffusion term D) with near-wall extensions (embodied in the matrix A_{ij})

$$\mathrm{d}X_i = U_i\,\mathrm{d}t + \sqrt{2\nu}\,\mathrm{d}W_i^X, \tag{34}$$

$$\mathrm{d}U_i = -\frac{\partial\langle P\rangle}{\partial x_i}\,\mathrm{d}t - (G_{ij} + A_{ij})(U_j - \langle U_j\rangle)\,\mathrm{d}t - \frac{1}{2}\frac{\epsilon}{k}(U_i - \langle U_i\rangle)\,\mathrm{d}t + \sqrt{D}\,\mathrm{d}W_i; \tag{35}$$

above, P is the kinematic pressure, $\mathrm{d}W_i$ is an increment of the Wiener process. The matrix G_{ij} is determined non-locally with the elliptic relaxation idea (Durbin, 1993). Further details and rationale of the present near-wall modelling are given in the paper of Wacławczyk et al. (2004). Moreover, the mean continuity equation and the constraint of constant fluid density are satisfied by the solution of two additional pressure correction equations of the elliptic type, as described by Minier and Pozorski (1999).

To deal with the heat transfer case, the near-wall model (Eqs. (34) and (35)), needs to be supplemented by a scalar evolution equation. In the PDF modelling with wall function approach (Pozorski et al., 2003a), we used the IEM closure (15). The question now is whether this model can also be used in the near-wall modelling (possibly with some modifications). From the stochastic system (34), (35), and (15), transport equations for the mean temperature $\langle T\rangle$, the fluctuating temperature variance $\langle\theta^2\rangle$ and the turbulent heat flux $\langle u_i\theta\rangle$ can be derived using standard methods. These equations are not shown here; yet, as a consequence of the random walk term with the transport coefficient ν in (34), inconsistencies will appear in the resulting transport equations for the statistical moments unless $Pr = \nu/\alpha = 1$. The problem of differing molecular transport coefficients for momentum and energy could have been overcome, if two separate sets of "velocity particles" and "temperature particles" had been used. Yet, the advantage of the joint U–T formulation would have been lost, since no joint velocity–temperature statistics would have been available.

Generally speaking, in the near-wall modelling of passive scalars, the molecular thermal diffusivity α should appear explicitly in transport equations. This causes difficulties in the Lagrangian setting, where molecular effects are usually introduced via random walk in stochastic particle positions. Mazumder and Modest (1997) proposed a modified near-wall model for temperature in the case of $Pr \neq 1$. Supplementary RHS terms are introduced to yield the correct transport equation for $\langle T\rangle$. However, it can be shown that the molecular diffusion terms in resulting mean equations for $\langle\theta^2\rangle$ and $\langle u_i\theta\rangle$ remain incorrect.

Here, our aim is to improve the existing proposal to account correctly for the terms with α in both $\langle T\rangle$ and $\langle\theta^2\rangle$ equations for the general case $Pr \neq 1$. The term with the Brownian motion in Eq. (34) already contains the kinematic viscosity ν that further enters the corresponding Eulerian equations for the mean temperature $\langle T\rangle$ and temperature variance $\langle\theta^2\rangle$, unless we add cor-

recting terms to the mixing model. After such a correction, the Laplacians in $\langle T\rangle$ and $\langle\theta^2\rangle$ equations are multiplied by α. A new formula for temperature increment is proposed as the simple IEM mixing model with a near-wall modification:

$$\mathrm{d}T = -\frac{1}{\tau_\phi}(T-\langle T\rangle)\,\mathrm{d}t - A(T-\langle T\rangle)\,\mathrm{d}t + B(T-\langle T\rangle)\,\mathrm{d}t + (\alpha-\nu)\frac{\partial^2\langle T\rangle}{\partial x_k\,\partial x_k}\,\mathrm{d}t, \tag{36}$$

where

$$A = \frac{\nu}{\langle\theta^2\rangle}\frac{\partial\langle T\rangle}{\partial x_k}\frac{\partial\langle T\rangle}{\partial x_k}, \quad B = \frac{\alpha-\nu}{2\langle\theta^2\rangle}\frac{\partial^2\langle\theta^2\rangle}{\partial x_k\,\partial x_k}. \tag{37}$$

The coefficient A counteracts the destruction term connected with the mean temperature while coefficient B corrects the Laplacian term which would appear in the resulting equation for the temperature variance. The last RHS term of Eq. (36) corrects the Laplacian in the mean temperature balance. However, the correcting terms involve second order derivatives of the mean fields; this impairs the accuracy of computations due to unavoidable statistical errors.

Again, integration of the PDF transport equation equivalent to Eqs. (34)–(36) yields evolution of the statistical moments. The mean temperature balance is correct and identical with Eq. (2). Resulting equation for the temperature variance

$$\frac{\partial\langle\theta^2\rangle}{\partial t} + \langle U_i\rangle\frac{\partial\langle\theta^2\rangle}{\partial x_i} = \alpha\frac{\partial^2\langle\theta^2\rangle}{\partial x_k\,\partial x_k} - \frac{\partial\langle u_i\theta^2\rangle}{\partial x_i} - 2\langle u_i\theta\rangle\frac{\partial\langle T\rangle}{\partial x_i} - C_\phi\langle\omega\rangle\langle\theta^2\rangle, \tag{38}$$

corresponds to an Eulerian model of passive scalar transport, cf. also exact Eq. (7). Apart from the modelled destruction of temperature fluctuations, all terms are exact. The transport equation for turbulent fluxes writes:

$$\frac{\partial\langle u_i\theta\rangle}{\partial t} + \langle U_k\rangle\frac{\partial\langle u_i\theta\rangle}{\partial x_k} = -\frac{\partial\langle u_iu_k\theta\rangle}{\partial x_k} - \left(\langle u_k\theta\rangle\frac{\partial\langle U_i\rangle}{\partial x_k} + \langle u_iu_k\rangle\frac{\partial\langle T\rangle}{\partial x_k}\right) + \nu\frac{\partial^2\langle u_i\theta\rangle}{\partial x_k\,\partial x_k} - G_{ik}\langle u_k\theta\rangle - \frac{1}{2}\left(\frac{\epsilon}{k} + C_\phi\langle\omega\rangle\right)\langle u_i\theta\rangle + 2\nu\frac{\partial\langle U_i\rangle}{\partial x_k}\frac{\partial\langle T\rangle}{\partial x_k} - A_{ik}\langle u_k\theta\rangle - A\langle u_i\theta\rangle. \tag{39}$$

This equation contains the Laplacian of $\langle u_i\theta\rangle$ multiplied by the kinematic viscosity ν instead of the factor $(\nu+\alpha)/2$ appearing in the exact turbulent heat flux balance (cf. Durbin, 1993). Besides, the last line of Eq. (39) contains three additional spurious terms that in general case do not sum up to 0. For this reason, the present modelling of molecular transport effects for $Pr \neq 1$ is regarded as being provisory.

The question arises whether the near-wall effects influence also turbulent heat fluxes and whether these effects can be modelled by elliptic relaxation method, by analogy to the procedure developed for turbulent stresses. It was checked by Durbin (1993) that computations with the elliptic model for redistribution term vary little from a simple local model. In our computations, however, G_{ik} is already modified by elliptic relaxation. As we expect, it should not influence the solution considerably.

5. Computation results for turbulent flow in heated channel

The PDF computation results are validated for the fully-developed channel flow with constant heat flux. For this case, usually 10^5 particles were used, and about 400 auxiliary cells (for the computation of statistics) in the wall-normal direction. Typically, 10^4 time steps were

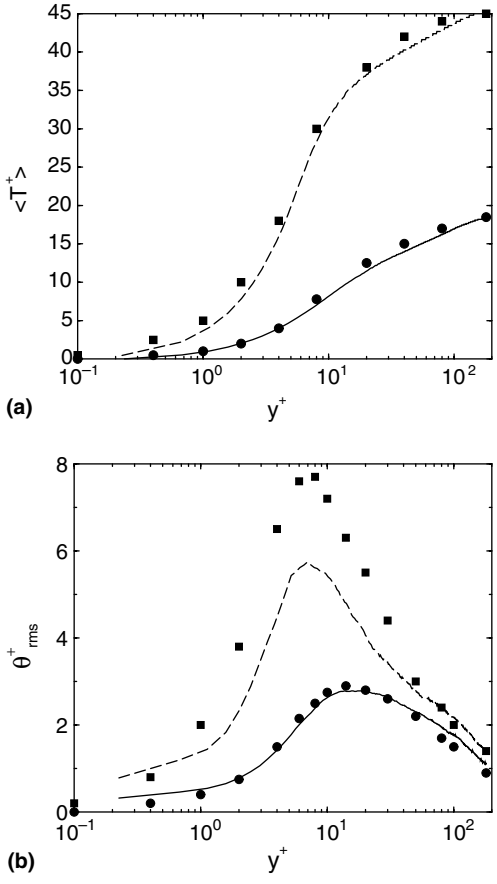

Fig. 1. Heated turbulent channel flow at $Re_\tau = 180$ with the isothermal wall boundary condition: (a) the mean temperature profile; (b) the temperature variance. DNS data of Tiselj et al. (2001) at $Pr = 1$ (●) and $Pr = 5.4$ (■). Scalar PDF results: lines.

necessary to achieve convergence. This corresponds to about 10^3 CPU seconds (on a Pentium III PC) for the scalar PDF variant and one order of magnitude more for the joint velocity–scalar computations. Independence of results with respect to the number of particles, the size of auxiliary mesh, and the time step has been checked. Final results have been gathered as a time average over, typically, last 1000 simulation time steps, once convergent solution has been obtained.

5.1. Results of scalar PDF approach

As a particular case of study for the scalar PDF variant, we have chosen the DNS computations reported by Kawamura et al. (1998) and Tiselj et al. (2001) at $Re_\tau = 180$. The computational results compared with those coming from the DNS study serve to assess the adequacy of modelling. In particular, they provide an a posteriori justification for the gradient hypothesis and the mixing model applied. The Prandtl number is a material constant, taken here as $Pr = 1$ and 5.4. For the sake of completeness, let us note that Lu and Hetsroni (1995) report DNS results for the isoflux heated channel at the same Re_τ and $Pr = 0.72$.

In the scalar PDF model, we have computed the temperature field using the input velocity statistics from the DNS data of Moser et al. (1999). In particular, the ν_t profile given by (6) and the mixing time scale τ_ϕ described by (16) have been used. For the isothermal wall condition, a constant profile of $Pr_t = 1$ has been taken. For the isoflux wall, a kind of "damping function", Eq. (22), has been assumed.

Temporal evolution of the first- and second-order moments, averaged over the channel height, provides a convergence criterion. It was satisfied after about 8000–16 000 iterations due to a considerable disparity of time scales in the core of the flow and in the near-wall region, respectively.

Predicted temperature statistics resulting from the PDF computation for the isothermal wall are shown in Fig. 1. Results for the isoflux wall boundary condition are shown in Fig. 2. For both cases, the sensitivity to the Prandtl number is examined; temperature statistics are in a fair agreement with the DNS. As readily noticed

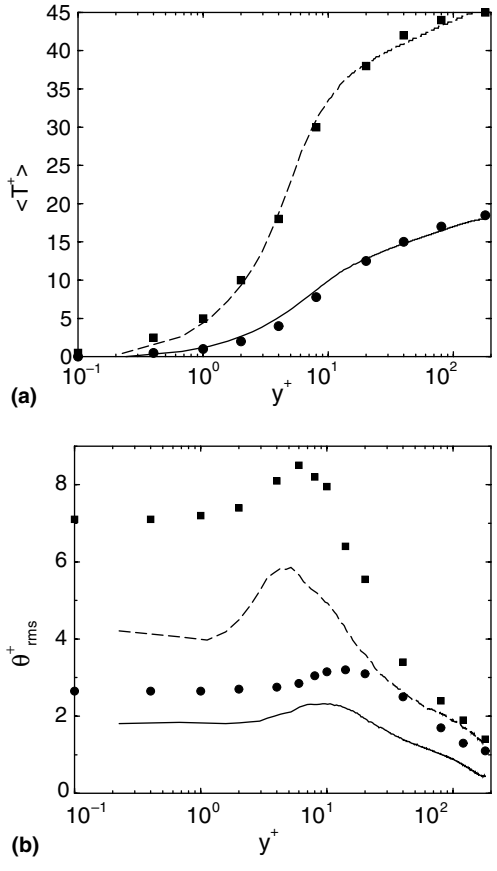

Fig. 2. Heated turbulent channel flow at $Re_\tau = 180$ with the isoflux wall boundary condition: (a) the mean temperature profile; (b) the temperature variance. DNS data of Tiselj et al. (2001) at $Pr = 1$ (●) and $Pr = 5.4$ (■). Scalar PDF results: lines.

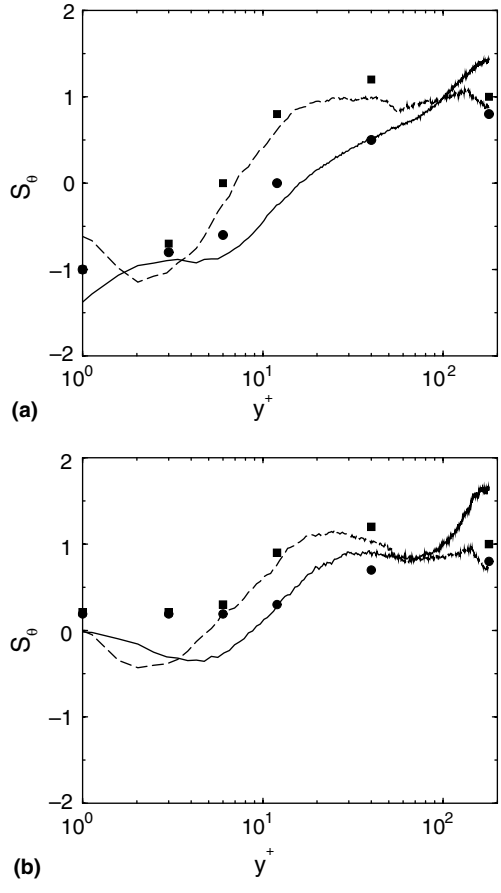

Fig. 3. Heated turbulent channel flow at $Re_\tau = 180$, the temperature skewness: (a) the isothermal case; (b) the isoflux case. DNS data of Tiselj et al. (2001) at $Pr = 1$ (●) and $Pr = 5.4$ (■). Scalar PDF results: lines.

from comparison of plots in Fig. 1(a) and plots in Fig. 2(a), the type of the temperature boundary condition (isothermal vs. isoflux wall) does not visibly influence the profiles of the mean temperature. As documented in the literature (Sommer et al., 1994; Kong et al., 2000; Tiselj et al., 2001), the thermal boundary conditions do have an impact on the level of near-wall temperature fluctuations. This is also seen in the present PDF computation results. Yet, as noticed in Fig. 2(b), the temperature fluctuations in the isoflux case are overdamped in the viscous flow region, specially for higher Pr. At the moment we have no explanation for this discrepancy. A reason might be the inadequacy of the mixing model constant C_ϕ: as documented in the DNS, the ratio of dynamical to thermal time scales varies substantially as a function of y^+ and Pr. Higher-order moments of the temperature distribution, such as the skewness and flatness, as well as the one-point PDFs (not shown) are also available; this is an a priori advantage of the PDF method over RANS. The temperature skewness is reported in Fig. 3 for both isothermal and isoflux cases and is again in a good agreement with the DNS.

5.2. Results of joint velocity–scalar PDF approach

The joint PDF computation results are also validated for the heated channel flow. As a case of study, we have taken the DNS of Kawamura et al. (1999) with $Pr = 0.71$, performed at $Re_\tau = 395$ (arguably the highest Re_τ available to date). The joint velocity–scalar PDF algorithm and program of the authors have been further developed for the present computation with the down-to-the-wall integration including elliptic relaxation (Wacławczyk et al., 2004). With this approach, the accuracy of the simpler scalar PDF method can be validated and additional statistics (such as the turbulent heat flux) can be obtained.

Results at $Re_\tau = 395$ for the mean temperature profile and the r.m.s. of temperature fluctuations $\langle\theta^2\rangle^{1/2}$ for the isothermal and isoflux wall b.c. are shown in Fig. 4. The mean temperature profile is identical for both cases; it is very well predicted in the near-wall region, less so is the temperature fluctuation level. It is too large for the isothermal wall and probably also for the isoflux wall; according to Tiselj et al. (2001), the temperature r.m.s. at the wall depends weakly on Re (they report $\theta_w^+ = 2.0$ at $Re_\tau = 170$). Moreover, the mean temperature is

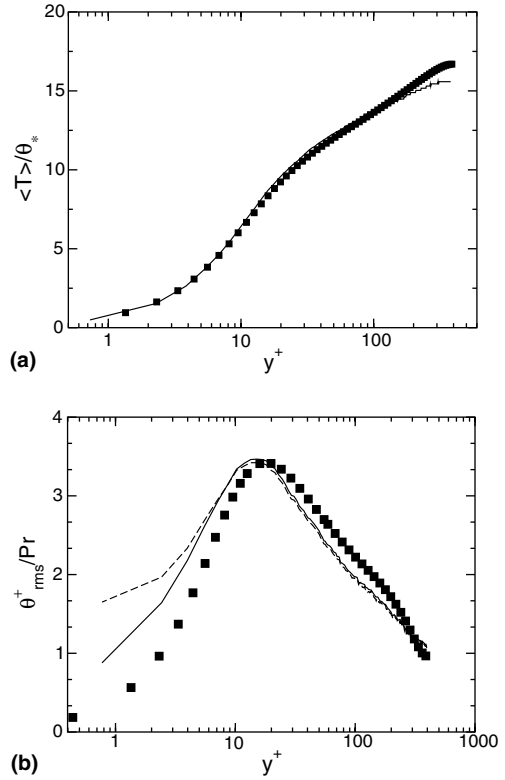

Fig. 4. Heated turbulent channel flow at $Re_\tau = 395$ and $Pr = 0.7$: (a) the mean temperature profile; (b) the fluctuating temperature r.m.s. Symbols: DNS data of Kawamura et al. (1999); lines: joint velocity–scalar PDF computation (solid line: isothermal wall, dashed line: isoflux wall).

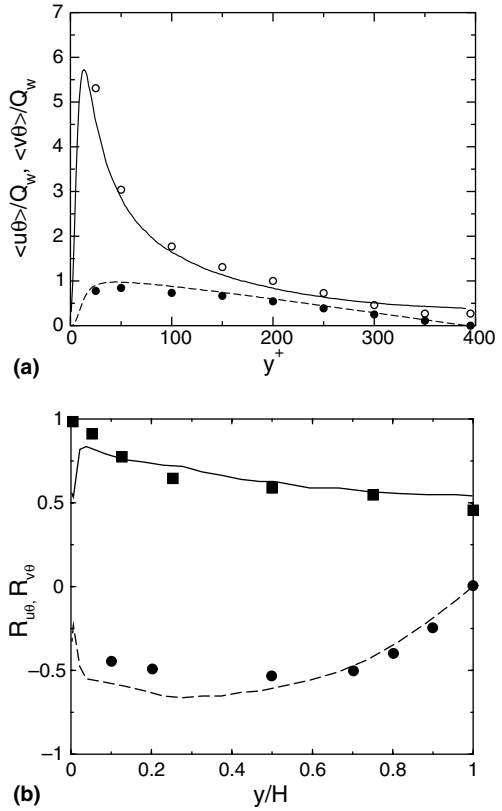

Fig. 5. Heated turbulent channel flow at $Re_\tau = 395$: (a) the heat flux components; (b) temperature–velocity correlation coefficients. Joint velocity–scalar PDF results: lines; DNS data of Kawamura et al. (1999): symbols. Streamwise components ($\langle u\theta\rangle, R_{u\theta}$): solid line and (■); wall-normal components ($\langle v\theta\rangle, R_{v\theta}$): dashed line and (●).

slightly underpredicted in the core region of the channel; this might be due to the IEM scalar mixing model used (cf. Pozorski et al., 2003a). The computed turbulent heat flux components $\langle u\theta\rangle$, $\langle v\theta\rangle$ are shown in Fig. 5(a). The wall-normal component agrees quite well with the DNS; so does the streamwise component that is now much better predicted than by the joint PDF method in the wall-function approach (Pozorski et al., 2003a). The temperature–velocity correlation coefficients $R_{u\theta}, R_{v\theta}$ are also available from the PDF computation. As seen in Fig. 5(b), the limit value of $R_{v\theta} = 0$ at the channel centerline is predicted correctly. The overall agreement of the profiles is reasonable, except close to the wall where it is flawed with numerical inaccuracies due to the 0/0 type limit behaviour (normally, we expect $R_{u\theta} = 1$ at the wall).

6. Conclusion

New modelling proposals have been advanced in the paper in the context of the PDF method for turbulent flows with the temperature field. First, a stochastic model for temperature (with dynamic variables assumed known) has been extended to account for the molecular thermal diffusivity that affects the temperature statistics in the near-wall region. Scalar PDF computation results demonstrate a fair agreement with the available DNS data for a turbulent channel flow with heated walls. The fluctuations are also sensitive to the molecular Prandtl number. In this first variant, the velocity PDF is not solved for, hence no joint statistics (such as the turbulent heat flux) are available. Both isothermal and isoflux boundary conditions have been considered.

Concerning the second variant, i.e. the joint velocity–scalar PDF method, the molecular transport effects in the near-wall region are explicitly accounted for and the resulting equations for the mean temperature and the thermal fluctuation variance are correct also for the Prandtl number different from unity; yet, some spurious terms exist in the modelled turbulent heat flux equations. The difficulties with a sound modelling of $Pr \neq 1$ originate in the Brownian motion (34) added to the convection of fluid elements; further work on this topic is warranted. The computation results for the heated channel flow are found to agree favourably with the DNS reference data.

Obviously, one-point statistical closures, including RANS models (such as k–ϵ–k_θ–ϵ_θ) and the PDF approach presented here, can yield the profile of $\langle\theta^2\rangle$, but are otherwise unable to provide the temperature spectrum. Some structural turbulence models have to be developed instead, e.g., along the lines of the unsteady 2D model with streamwise streaky structures (Kasagi et al., 1989) or the scalar filtered density function (FDF) model combined with the dynamics of the large-eddy velocity modes (such as POD) in the near-wall region (Wacławczyk and Pozorski, 2004). Moreover, the isothermal and isoflux wall boundary conditions considered here represent two limit cases of a more general conjugate heat transfer where the level of near-wall thermal fluctuations is influenced directly by the wall material characteristics. Work along these lines are in progress.

References

Dahlström, S., Davidson, L., 2003. Hybrid RANS/LES employing interface condition with turbulent structure. In: Hanjalić, K., Nagano, Y., Tummers, M. (Eds.), Turbulence, Heat and Mass Transfer, vol. 4. Elsevier, Amsterdam. pp. 689–696.

Dopazo, C., 1994. Recent developments in PDF methods. In: Libby, P.A., Williams, F.A. (Eds.), Turbulent Reacting Flows. Academic Press, New York, pp. 375–474.

Durbin, P.A., 1993. A Reynolds-stress model for near-wall turbulence. J. Fluid Mech. 249, 465–498.

Geshev, P.I., 1978. The influence of thermal conductivity of the wall on the value of the turbulent Prandtl number in a viscous sublayer. Inzhenerno-Fizicheskij Zhurnal 35, 292–296 (in Russian).

Hamba, F., 2002. An approach to hybrid RANS/LES calculation of channel flows. In: Rodi, W., Fueyo, N. (Eds.), Engineering Turbulence Modelling and Measurements, vol. 5. Elsevier, Amsterdam. pp. 297–305.

Jones, W.P., 2002. The joint scalar probability density function. In: Launder, B.E., Sandham, N.D. (Eds.), Closure Strategies for Turbulent and Transitional Flows. University Press, Cambridge, pp. 582–625.

Kasagi, N., Kuroda, A., Hirata, M., 1989. Numerical investigation of near-wall turbulent heat transfer taking into account the unsteady heat conduction in the solid wall. J. Heat Transfer 111, 385–392.

Kawamura, H., Abe, H., Matsuo, Y., 1999. DNS of turbulent heat transfer in channel flow with respect to Reynolds and Prandtl number effects. Int. J. Heat Fluid Flow 20, 196–207.

Kawamura, H., Ohsaka, K., Abe, H., Yamamoto, K., 1998. DNS of turbulent heat transfer in channel flow with low to medium-high Prandtl number fluid. Int. J. Heat Fluid Row 19, 482–491.

Kays, W.M., 1994. Turbulent Prandtl number—where are we? J. Heat Transfer 116, 285–295.

Kim, J., Moin, P., 1989. Transport of passive scalars in a turbulent channel flow. In: Turbulent Shear Flows, vol. VI. Springer, pp. 85–96.

Kong, H., Choi, H., Lee, J.S., 2000. Direct numerical simulation of turbulent thermal boundary layers. Phys. Fluids 12, 2555–2568.

Launder, B.E., 1988. On the computation of convective heat transfer in complex turbulent flows. J. Heat Transfer 110, 1112–1128.

Lu, D.M., Hetsroni, G., 1995. Direct numerical simulation of a turbulent open channel flow with passive heat transfer. Int. J. Heat Mass Transfer 38, 3241–3251.

Mazumder, S., Modest, M.F., 1997. A stochastic Lagrangian model for near-wall turbulent heat transfer. J. Heat Transfer 119, 46–52.

Minier, J.P., Pozorski, J., 1999. Wall boundary conditions in PDF methods and application to a turbulent channel flow. Phys. Fluids 11, 2632–2644.

Moser, R.D., Kim, J., Mansour, N.N., 1999. Direct numerical simulation of turbulent channel flow up to $Re_\tau = 590$. Phys. Fluids 11, 943–945.

Nagano, Y., 2002. Modelling heat transfer in near-wall flows. In: Launder, B.E., Sandham, N.D. (Eds.), Closure Strategies for

Turbulent and Transitional Flows. University Press, Cambridge, pp. 188–247.

Piomelli, U., Balaras, E., 2002. Wall-layer models for large-eddy simulations. Annu. Rev. Fluid Mech. 34, 349–374.

Polyakov, A.F., 1974. Wall effect on temperature fluctuations in the viscous sublayer. Teplofizika Vysokikh Temperatur 12, 328–337 (in Russian).

Pope, S.B., 2000. Turbulent Flows. Cambridge University Press.

Pozorski, J., Wacławczyk, M., Minier J.P, 2003a. Full velocity–scalar probability density function computation of heated channel flow with wall function approach. Phys. Fluids 15, 1220–1232.

Pozorski, J., Wacławczyk, M., Minier, J.P., 2003b. Probability density function computation of heated turbulent channel flow with the bounded Langevin model. J. Turbul. 4, art. 011, pp. 1–23.

Sommer, T.P., So, R.M.C., Zhang, H.S., 1994. Heat transfer modeling and the assumption of zero wall temperature fluctuations. J. Heat Transfer 116, 855–863.

Tiselj, I., Pogrebnyak, E., Li, C., Mosyak, A., Hetsroni, G., 2001. Effect of wall boundary condition on scalar transfer in a fully developed turbulent flume. Phys. Fluids 13, 1028–1039.

Wacławczyk, M., Pozorski, J., 2004. Conjugate heat transfer modelling using the FDF approach for near-wall scalar transport coupled with the POD method for flow dynamics. In: Andersson, H.I., Krogstadt, P.A. (Eds.), Advances in Turbulence X. CIMNE, Barcelona.

Wacławczyk, M., Pozorski, J., Minier, J.P., 2004. PDF computation of turbulent flows with a new near-wall model. Phys. Fluids 16, 1410–1422.